THE FLOW OF A NONEQUILIBRIUM IONIZED RADIATING GAS AROUND A BODY WITH CONSIDERATION OF TEMPERATURE DIFFERENCE BETWEEN ELECTRONS AND IONS

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The flow around a blunt body at hypersonic speed by a current of nonequilibrium ionized monatomic nonviscous radiating gas is studied, with consideration of temperature difference between the electron gas and the ion-atom gas. Atomic excitation due to collisions with electrons and subsequent ionization, as well as photoionization, are taken into consideration. Since the value of the shock wave separation is small in comparison with the characteristic dimension of the body, the radiation transfer equation is written in the local one-dimensional planar layer approximation. The influence of incident flow parameters upon the flow field across the shock wave and the distribution of radiation thermal flux are studied.

Terminology:

r, radius vector, calculated from the center of curvature of the body about which flow occurs; $\mathbf{r}_T$, $\mathbf{r}_b$, radius vectors of the body surface and shock wave; ε, separation of the shock wave $\varepsilon = r_b - r_T$; L, characteristic dimension of the body; **V**, W, vector and modulus of total gas velocity, respectively; W_m, maximum gas velocity (velocity of escape into a vacuum); u, component of gas velocity along the radius vector; p, gas pressure; ρ, gas density; ρ_i, ion density; α, degree of nonequilibrium gas ionization, $\alpha = \rho_i/\rho$; α_E, degree of equilibrium gas ionization; T_a, temperature of atoms and ions, T_e, electron temperature; T_{e*}, excitation temperature; T_m, body surface temperature; A, atom in the fundamental state; A^*, atom in an excited state; ν, frequency; ν_j, T_j, ionization frequency and temperature; m_a, mass of an atom; m_e, e, electronic mass and charge; C_E, electron-atom excitation cross section; n_{ea}, ionization reaction velocity for electron–atom shock; n_e, electron density; R, specific gas constant; K, Boltzmann constant; E, internal energy per unit mixture mass; $\varkappa_\nu$, spectral absorption coefficient per unit mass of monatomic gas; δ, blackness coefficient of body surface; τ_ν, optical coordinate; μ_i, mean cosine of angle for given angular zone of photon propagation; I_ν, radiation spectral intensity; $B_\nu(T_e)$, Planck function; $\mathbf{q}_\nu$, **q**, spectral and total radiant energy flux vectors $\mathbf{q} = \int_0^\infty \mathbf{q}_\nu d\nu$; M, Mach number; ∞, b, indices related to gas parameters in undisturbed flow region and in shock wave, respectively.

1. Fundamental System of Equations. In order to describe the nonequilibrium processes in the shock layer it is necessary to define a certain kinetic model for the gas. In this study the kinetics proposed for argon by Chaplin in [1] will be used. Consideration is made of reactions

$$A + e \rightleftarrows A^* + e, \quad A^* + e \rightleftarrows A^+ + e, \quad A + h\nu \rightleftarrows A^+ + e$$

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The fundamental system of equations describing the flow of gas in the shock layer contains the following equations: the continuity equation, the motion and energy equations

$$\nabla \cdot (\rho \mathbf{V}) = 0 \tag{1.1}$$

$$(\mathbf{V} \cdot \nabla) \mathbf{V} = -\frac{1}{\rho} \nabla p \tag{1.2}$$

$$\nabla \cdot \left[\rho \mathbf{V} \left(\frac{1}{2} V^2 + E + \frac{1}{\rho}\right)\right] = -\nabla \cdot \mathbf{q} \tag{1.3}$$

and the electron temperature equation

$$\nabla \cdot \left(\frac{5}{2} K T_e n_e \mathbf{V}\right) = \sum_i Q_i \tag{1.4}$$

Herein Q_i is the contribution of the i-th process to the electron energy balance. The following problems are considered: Q_1, elastic electron-ion A^+ collisions; Q_2, losses to atom ionization A; Q_3, contribution of the photoionization reaction to electron energy:

$$Q_1 = \frac{n_e^2 e^4}{m_a} \left(\frac{8\pi m_e}{K T_e}\right)^{1/2} \left(\frac{T_a}{T_e} - 1\right) \ln\left[\frac{9}{4} \frac{(K T_e)^3}{\pi n_e e^6} + 1\right] \tag{1.5}$$

$$Q_2 = -K T_j n_{ea}, \quad Q_3 = -K T_e \nabla \cdot \int_{\nu_j}^{\infty} (h\nu)^{-1} \mathbf{q}_\nu d\nu \tag{1.6}$$

The equation of nonequilibrium ionization velocity is

$$\nabla \cdot (\rho \mathbf{V} \alpha) = m_a n_{ea} - m_a \nabla \cdot \int_{\nu_j}^{\infty} (h\nu)^{-1} \mathbf{q}_\nu d\nu \tag{1.7}$$

According to [1],

$$n_{ea} = \left(\frac{\rho}{m_a}\right)^2 2 C_E \left(\frac{2}{\pi m_e}\right)^{1/2} (K T_e)^{3/2} \left(\frac{T_{e*}}{T_e} + 2\right) \exp\left(-\frac{T_{e*}}{T_e}\right) \alpha (\alpha_E - \alpha) \tag{1.8}$$

The equation of state is given by

$$p = \rho R (T_a + \alpha T_e) \tag{1.9}$$

The equation of energy transfer in direction s is

$$\frac{\partial I_\nu}{\partial s} = \rho (1 - \alpha) \varkappa_\nu \left(\frac{1 - \alpha_E}{1 - \alpha} B_\nu - I_\nu\right) \tag{1.10}$$

The boundary conditions for the system of Eqs. (1.1)-(1.4), (1.7), (1.9) are analogous to the boundary conditions on the body and in the shock wave [2], excepting the condition of conservation of energy and the supplementary conditions on the continuity of T_e upon transition across the shock wave

$$\frac{5}{2} R (T_{a\infty} + \alpha_\infty T_{e\infty}) + \alpha_\infty R T_j + \frac{W_\infty^2}{2} + \frac{q_b}{\rho_\infty W_\infty}$$
$$= \frac{5}{2} R (T_{ab} + \alpha_\infty T_{eb}) + \alpha_\infty R T_j + \frac{W_b^2}{2} + \frac{q_b}{\rho_b W_b} \tag{1.11}$$

$$T_{e\infty} = T_{eb} \tag{1.12}$$

Having performed computations similar to those of [2], we obtain an expression for the radiant energy flux and its divergence in the local one-dimensional planar layer approximation,

$$q(\tau_\nu) = 2\pi \sum_{i=0}^{N-1} (\mu_{i+1} - \mu_i) [F_1^i(\tau_\nu) + \delta Q_2^i(\tau_\nu) + (1 - \delta) F_2^i(\tau_\nu) + F_3^i(\tau_\nu)] \tag{1.13}$$

$$\nabla \cdot \mathbf{q} = 2\pi\rho (1 - \alpha) \varkappa_\nu \sum_{i=0}^{N-1} \left(\frac{\mu_{i+1} - \mu_i}{\mu_i}\right) [-F_1^i(\tau_\nu) - \delta Q_2^i(\tau_\nu) \mu_i -$$
$$- (1 - \delta) F_2^i(\tau_\nu) - F_3^i(\tau_\nu)] + 4\pi\rho (1 - \alpha) \varkappa_\nu Q_1(\tau_\nu) \tag{1.14}$$

Herein

$$F_1^i(\tau_\nu) = \exp\left(-\frac{\tau_\nu}{\mu_i}\right)\int_0^{\tau_\nu} \frac{1-\alpha_E}{1-\alpha} B \exp\left(\frac{t}{\mu_i}\right) dt \tag{1.15}$$

$$F_2^i(\tau_\nu) = \exp\left(-\frac{\tau_\nu}{\mu_i}\right)\int_0^{\tau_{\nu b}} \frac{1-\alpha_E}{1-\alpha} B \exp\left(-\frac{t}{\mu_i}\right) dt \tag{1.16}$$

$$F_3^i(\tau_\nu) = \exp\left(\frac{\tau_\nu}{\mu_i}\right)\int_0^{\tau_{\nu b}} \frac{1-\alpha_E}{1-\alpha} B \exp\left(-\frac{t}{\mu_i}\right) dt \tag{1.17}$$

$$Q_2^i(\tau_\nu) = B(T_m)\exp\left(-\frac{\tau_\nu}{\mu_i}\right) \tag{1.18}$$

$$Q_1(\tau_\nu) = \frac{1-\alpha_E}{1-\alpha} B \tag{1.19}$$

$$B = \int_{\nu_j}^{\nu_k} B_\nu d\nu = \frac{2K^4T_e^4}{h^3c^2}\left\{\exp\left(-\frac{h\nu_j}{KT_e}\right)\left[\left(\frac{h\nu_j}{kT_e}\right)^3 + 3\left(\frac{h\nu_j}{KT_e}\right)^2 + 6\left(\frac{h\nu_j}{KT_e}\right) + 6\right] - \right.$$
$$\left. - \exp\left(-\frac{h\nu_k}{KT_e}\right)\left[\left(\frac{h\nu_k}{KT_e}\right)^3 + 3\left(\frac{h\nu_k}{KT_e}\right)^2 + 6\left(\frac{h\nu_k}{KT_e}\right) + 6\right]\right\} \tag{1.20}$$

2. Flow along a Null Current Line. The study of Eqs. (1.1)-(1.4), (1.7), (1.9), and (1.10) presents great difficulties, and so as a first step in the attainment of a solution for the entire infrasonic region we will examine the gas flow along a null current line. In this case the system of partial differential equations simplifies to a system of regular differential equations. To close the system, values were set for velocity profile $u=u_b\xi$, where $\xi=(r-r_T)/(r_b-r_T)$, and the dimensionless shock wave separation

$$\varepsilon = 0.775/\rho^* \qquad \left(\rho^* = \int_0^1 \rho(\xi)/\rho_\infty d\xi\right)$$

We introduce the dimensionless quantities

$$u^* = \frac{u}{W_m}, \quad \rho^* = \frac{\rho}{\rho_\infty}, \quad p^* = \frac{p}{\rho_\infty W_m^2}, \quad T_a^* = \frac{RT_a}{W_m^2}, \quad T_e^* = \frac{RT_e}{W_m^2}$$
$$q^* = \frac{2q}{\rho_\infty W_m^3}, \quad r^* = \frac{r}{L}$$

In this case the fundamental system of equations describing the flow of a nonequilibrium ionized radiating gas along a null current line near a spherical body takes on the form (asterisks have been omitted for the dimensionless quantities)

$$\rho u \frac{du}{d\xi} = -\frac{dp}{d\xi} \tag{2.1}$$

$$\rho u \frac{d}{d\xi}\left[5(T_a + \alpha T_e) + 2\alpha T_j + u^2\right] = -\frac{dq}{d\xi} \tag{2.2}$$

$$\rho u \frac{d}{d\xi}\left(\frac{5}{2}\alpha T_e\right) = \varepsilon(Q_1 - T_j m_a n_{ea}) - T_e m_a \frac{d}{d\xi}\int_{\nu_j}^{\infty} (h\nu)^{-1} q_\nu d\nu \tag{2.3}$$

$$\rho u \frac{d\alpha}{d\xi} = \varepsilon m_a n_{ea} - m_a \frac{d}{d\xi}\int_{\nu_j}^{\infty} (h\nu)^{-1} q_\nu d\nu \tag{2.4}$$

$$p = \rho(T_a + \alpha T_e) \tag{2.5}$$

$$\mu \frac{dI_\nu}{d\xi} = \varepsilon\rho(1-\alpha)\varkappa_\nu\left(\frac{1-\alpha_E}{1-\alpha} B_\nu - I_\nu\right) \tag{2.6}$$

This system of differential equations was separated according to derivatives with respect to ξ and integrated from the shock wave ($\xi=1$) to the body ($\xi=0$) by the Euler method with multiple conversion with-

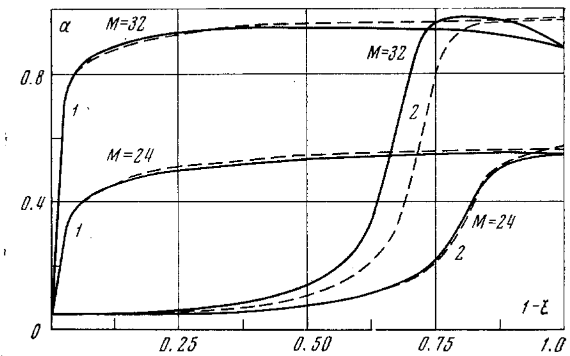

Fig. 1

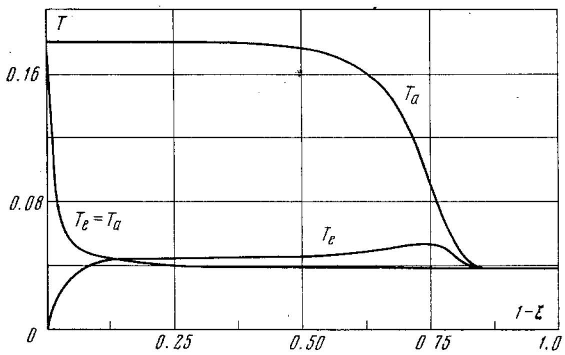

Fig. 2

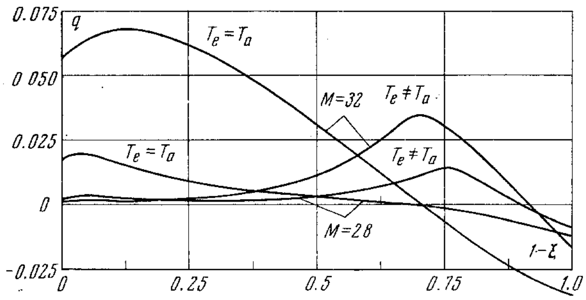

Fig. 3

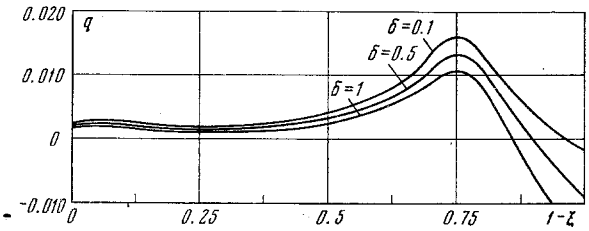

Fig. 4

out division of the integration step. Due to the integral character of the radiation terms the solution was obtained by the iteration method.

3. Evaluation of Results. We shall present the results of calculation for flow about a sphere of radius $R=4$ cm by a current of argon for $T_{a\infty}=300°K$, $T_{e\infty}=300°K$, $\alpha_b=5\cdot 10^{-2}$, $24 \leq M \leq 32$, $p_\infty=0.001$ atm. In the majority of calculations it was assumed that the blackness coefficient of the body surface $\delta=0.5$. Calculations were conducted for the general case $T_e \neq T_a$ and, for comparison, for $T_e=T_a$. (The results of calculations without consideration of radiation are indicated by dashed lines.)

Figure 1 presents the profiles of degree of ionization across the shock layer. It is evident that a sharp difference exists in the length of the relaxation zone for the cases $T_e=T_a$ (curve 1) and $T_e \neq T_a$ (curve 2), although the values of the degree of ionization on the body practically coincide. The increase in relaxation zone length for the case $T_e \neq T_a$ is explained by the fact that elastic energy transfer between the electronic and atomic-ionic components of the gas does not occur instantaneously, but continues over the course of a certain period of time. Consideration of radiation, as was noted in [2], leads to a certain decrease in the relaxation zone length.

Figure 2 presents the temperature profiles in the shock layer for $M=28$. In the case $T_e=T_a$ a sharp temperature drop to the equilibrium value near the shock layer is observed. (This is explained by the flow into this region of Townsend ionization.) In the case $T_e \neq T_a$ a significant drop in T_a is observed much later in accordance with the new position of the Townsend ionization front. Immediately beyond the shock wave a sharp increase in electron temperature from $T_{eb}=T_{e\infty}$ to some value exceeding the equilibrium value T_e takes place: this rise is characteristic of the intense energy exchange between ions and electrons due to elastic collisions. Further, with increase in T_e the energy loss of the electron gas to ionization increases, and T_e, having passed a maximum, begins to oscillate along with T_a down to equilibrium, where T_e and T_a coincide. The appearance of a maximum in T_e near the Townsend ionization front is due to the presence of two opposing factors: increase in electron gas energy due to elastic collisions, and energy loss to ionization.

Figure 3 presents radiation flux q profiles in the shock layer for various Mach numbers. There is a sharp difference in q profiles for the cases $T_e=T_a$ and $T_e \neq T_a$. In the case $T_e=T_a$ q has one maximum, the position of which coincides with the location of the Townsend ionization front. In the shock layer the value of q changes sign: near the shock wave q is positive, while near the body it is negative. In the case

$T_e \neq T_a$ q has two maxima; a fundamental maximum displaced toward the body (corresponding to the displacement of the Townsend ionization front for $T_e \neq T_a$), and a second maximum located near the shock wave. With an increase in Mach number both maxima are displaced toward the shock wave, while the first maximum increases, and the second decreases. For $T_e \neq T_a$ the radiation thermal flux on the body is significantly lower than for $T_e = T_a$.

Figure 4 presents the radiant flux profiles in the shock layer for $M = 28$ for various blackness coefficients of the body surface δ. With an increase in δ the value of the radiant flux on the body increases, since the body then reflects a smaller portion of the radiant flux back into the shock layer.

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